Unit BOG: Dwell…A Closed-Form Approach

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Abstract: In this paper we explore the Army's Active Component Brigade Combat Teams' (AC BCTs) unit *Boots on the Ground* (BOG) to *Dwell* ratio. In particular, we derive a closed-form result which is effectively independent of the underlying deployment schedule and deployment management technique. Using this result we show that the planned growth from 43 to 45 BCTs is insufficient to reach the Army's targeted 1:2 objective for AC BCTs.

Keywords: Army Force Generation (ARFORGEN), Brigade Combat Team (BCT), Boots on the Ground (BOG), dwell time

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Introduction and Background

When a member of the military is deployed overseas in a combat environment the oft-heard question is "When are you coming home?" Unfortunately, in the current operational environment, the question these deployed warriors hear upon their return is often "When do you have to go back?" It is no secret that the frequency of deployments has had a significant impact on the morale and well-being of many military members and their families. This is especially true for the officers and Soldiers in the US Army who have borne the brunt of the operational impact of the on-going conflicts. It is a priority amongst the US Army's senior leadership to implement policies and procedures to improve this situation.

The Army refers to the time a Soldier or Brigade Combat Team (BCT) spends deployed overseas in a combat environment as Boots on the Ground time, or BOG. Conversely, the time a Soldier or BCT spends between deployments is known as "dwell." This BOG:Dwell ratio is an important statistic which is tracked in-depth as it serves as a leading indicator of recruiting, retention and morale issues for the Army, its Soldiers, and their families. With the current BOG:Dwell ratio for deployed BCTs hovering between 1:0.85 and 1:1, it is not uncommon for a young Soldier to have spent nearly 50% of his time in service deployed in support of on-going operations (Department of the Army, 2008c).

Recognizing that these ratios are unsustainable, the Army has initiated programs and implemented policies aimed at improving these numbers. Most notably, the self-explanatory "Grow the Army" program (in conjunction with modularization) was designed to increase the number of Active Component Brigade Combat Teams (AC BCTs) from its current level of 43 to 48 – a number recently reduced to 45 by Secretary of Defense Robert Gates (Department of the Army, 2008b; Congressional Budget Office, 2009). Additionally, the Department of the Army (DA) has issued several policies designed to limit the BCT's BOG and protect its dwell. Taken as a whole, their stated objective is simple – improve the BOG:Dwell ratio to a reasonable level, namely 1:2 (Department of the Army, 2008a).

Concurrent with these programs and policies, the Army has recently implemented a new management support tool designed to fairly and equitably rotate units while providing the combatant commanders with the force necessary to meet our national security requirements. Dubbed Army Force Generation or ARFORGEN for short, it is a "structured progression of increased unit readiness over time resulting in recurring periods of availability of trained, ready, and cohesive units" (Department of the Army, 2007a). As a matter of design, ARFORGEN works the AC BCTs through three force pools designated as Reset (reorganizing), Train/Ready (preparing and available for unforeseen requirements), and Available (fully deployable) (Department of the Army, 2007a). However, in light of the current

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operational environment, these force pools effectively equate to not-deployed (Reset and Train/Ready) and deployed (Available).

A natural question arises: will it work? More specifically, once the Army reaches 45 BCTs, will the expanded Army along with its policies and application of ARFORGEN be able to reach a 1:2 BOG:Dwell ratio for AC BCTs? Can the Army restore its balance?

Unit BOG:Dwell

At its core, the Army's AC BCT BOG:Dwell ratio is a simple supply and demand issue, where supply is represented by the number of rotational AC BCTs and demand is the number required for deployment. Furthermore, when driven by a constant demand signal, this ratio converges to a theoretical limit, one which can be constructed in a very general way using basic algebra and calculus. Accordingly, in this section we will formally derive this ratio, analyze it, and develop simple, irrefutable facts about how well the Army can actually do under various scenarios.

With this in mind, we will use the following notation:

1. $N \equiv$ the Army's number of rotational, active component (AC) BCTs ($N \in \mathbb{N}$)
2. $M \equiv$ the sustained number of AC BCTs required to support the GWOT; alternatively, the number of AC BCTs in ARFORGEN's Available phase ($M \in \mathbb{N}$, $M \leq N$)
3. $x \equiv$ the standard duration of an AC BCT deployment (may be expressed in any convenient or desired time units (i.e., days, months, years, etc.)) ($x \in \mathbb{R}^+$)
4. $N \equiv$ the standard duration of an AC BCT's Relief in Place – Transfer of Authority (RIP-TOA); alternatively, the duration of the overlap between a deployed AC BCT and its replacement (must be expressed in the same time units as x) ($n \in \mathbb{R}^+$, $n < x$)
5. $T \equiv$ the length of the analysis period (must be expressed in the same time units as x) ($T \in \mathbb{R}^+$, $T > x - n$)

Theorem: As T tends to infinity, if $\frac{N}{M} > 1 + \frac{n}{x-n}$, the AC BCT BOG:Dwell ratio is given by the relation:

$$g(T) = 1 : \frac{N(x-n)}{Mx} - 1 \qquad (1)$$

Proof (by construction): Without loss of generality, we start by noting that the left-hand side of the Army's average AC BCT BOG:Dwell ratio during a T-day analysis period is simply the aggregate number of BOG days during T. With a requirement for M deployed BCTs throughout T, the left-hand side is minimally MT. Additionally, we note that in order for a BCT to be replaced at the termination of it x-day rotation, it must complete an n-day RIP-TOA cycle. Assuming that every RIP-TOA cycle begins n-days prior to the end of a deployed BCT's rotation, there are minimally $\lfloor T/(x-n) \rfloor$ overlaps for each of the $i = \{1, 2, \ldots, M\}$ BCTs required during T, where $\lfloor T/(x-n) \rfloor$ denotes the *floor function* and represents the integer portion of the expression $T/(x-n)$ following division. Finally, based on the Army's AC BCT deployment history prior to the beginning of the analysis period, it is possible that a full or partial RIP-TOA cycle terminates for an incoming BCT at some time t_i, where $t_i \in (0, x\text{-}n)$, $i = \{1, 2, \ldots, M\}$. We denote this potential overlap time as β_i, where $0 < \beta_i \leq n$. Accordingly, as we have a requirement for M BCTs, it follows that the total number of overlap days is $Mn \lfloor T/(x-n) \rfloor + \Sigma_{i=1}^{M} \beta_i$. Aggregating the above, the left-hand side of the AC BCT BOG:Dwell ratio is:

$$Total\ Number\ of\ BCT\ Deployed\ Days = MT + Mn \left\lfloor \frac{T}{x-n} \right\rfloor + \sum_{i=1}^{M} \beta_i \qquad (2)$$

In order to remove the floor function from (2), we can replace $\lfloor T/(x-n) \rfloor$ with the expression $\frac{T}{x\text{-}n} - \alpha$ where $\alpha \in [0, 1)$ and represents the decimal portion of $T/(x-n)$ following division. This substitution yields:

$$Total\ Number\ of\ BCT\ Deployed\ Days = MT + Mn \left(\frac{T}{x-n} - \alpha \right) + \sum_{i=1}^{M} \beta_i \qquad (3)$$

In a similar fashion, we note that the right-hand side of the ratio is simply the aggregate number of dwell days during T. According to Army and DoD policies, if a day is not counted as BOG, then it is dwell (Department of the Army, 2007b). As such the aggregate number of dwell days is given by:

$$\begin{aligned} Total\ Number\ of\ BCT\ Dwell\ Days &= Total\ Number\ of\ BCT\ Days \\ &\quad - Total\ Number\ of\ BCT\ Deployed\ Days \\ &= NT - MT - Mn\left(\frac{T}{x-n} - \alpha\right) - \sum_{i=1}^{M} \beta_i \end{aligned} \qquad (4)$$

Placing (3) and (4) into the BOG:Dwell ratio, treating x, n, N, M, α , and β_i as constants, and collecting terms with respect to T we obtain:

$$g(T) = \frac{T\left(M + \frac{Mn}{x-n}\right) - Mn\alpha + \sum_{i=1}^{M} \beta_i}{T\left(N - M - \frac{Mn}{x-n}\right) + Mn\alpha - \sum_{i=1}^{M} \beta_i} \qquad (5)$$

Taking the limit of $g(T)$ as $T \to \infty$ it is seen that $M + \frac{Mn}{x-n} > 0\ \forall\ M, n,$ and x, which implies the limit of the numerator as $T \to \infty$ is ∞. Similarly, in the denominator when $N - M - \frac{Mn}{x\text{-}n} > 0$ when $\frac{N}{M} > 1 + \frac{n}{x-n}$. Assuming this relation holds, then the limit of the denominator as $T \to \infty$ is ∞. With this in mind, we have the indeterminate form ∞/∞. Applying L'Hôpital's rule, we take the derivative of the numerator and the denominator with respect to T, which yields:

$$\lim_{T\to\infty} g(T) = \frac{M + \frac{Mn}{x-n}}{N - M - \frac{Mn}{x-n}} \qquad (6)$$

Multiplying the top and bottom of (6) by $(M + (x - n))^{-1}$ we get:

$$\lim_{T\to\infty} g(T) = \frac{1}{\frac{N}{M+\frac{Mn}{x-n}} - 1} \qquad (7)$$

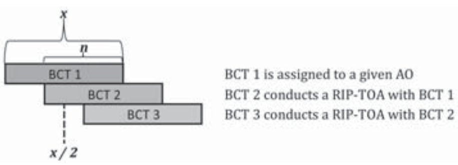
Finally, collecting terms with respect to M and simplifying produces:

$$\lim_{T\to\infty} g(T) = \frac{1}{\frac{N}{M(1+\frac{n}{x-n})} - 1}$$

$$= \frac{1}{\frac{N}{M(\frac{x-n}{x-n}+\frac{n}{x-n})} - 1}$$

$$= \frac{1}{\frac{N(x-n)}{Mx} - 1} \blacksquare$$

In the above argument we have derived the AC BCT BOG:Dwell ratio when $\frac{N}{M} > 1 + \frac{n}{x-n}$. Accordingly, we will take a moment to discuss the circumstances under which this relation holds, starting with reasonable values of n and x. For example, if $n \geq x/2$ then an incoming BCT would start its RIP-TOA cycle with the subsequent incoming BCT prior to or immediately after transitioning with the outgoing BCT. This dynamic is illustrated graphically in Figure 1 below.



From the above argument, it is clear that any reasonable selection for n should be less than $x/2$. With this in mind, if $n < x/2$, then $\frac{n}{x-n} < 1$ and the maximum value of $1+\frac{n}{x-n}$ is less than 2. Accordingly, if $\frac{N}{M} \geq 2$, then $N - M - \frac{Mn}{x-n} > 0$ and the limit of the denominator as $T \to \infty$ is ∞.

In fact, requiring n to be less than $x/2$ is not really a practical upper bound. Specifically, if n is one day less than x / 2, then a BCT would be alone it its AO for 2 days prior to starting its RIP-TOA cycle with the incoming BCT. Clearly, this is impractical, especially in light of our ongoing operations. Specifically, as it currently stands, deployments last 12 months with roughly 1.5 months of overlap. Using these as reasonable values for x and n respectively, we observe that $1 + \frac{n}{x-n}$ is approximately 1.14. With this in mind, if $\frac{N}{M} \geq 1.15$, then the theorem holds. In terms of our current force of 43 BCTs, this implies that the AC BCT BOG:Dwell ratio is 1: $\frac{N(x-n)}{Mx} - 1$ for $M \leq \frac{43}{1.15} = 37.3$. In short, the theorem is applicable to nearly all reasonable scenarios.

Implications

Using the above result, we can now explore what is theoretically possible once the Army reaches 45 AC BCTs, of which only 44 are rotational due to the permanent assignment of 1-2 ID on the Korean peninsula. Specifically, we fix N at 44 and n at 40 days respectively, and we calculate the AC BCT BOG:Dwell ratio for various combinations of M and x. The results of this analysis are seen in Figure 2 below.

As Figure 2 shows, with a demand of 16 BCTs and 1 year deployments, the Army's AC BCT BOG:Dwell ratio is 1.45 - 200 dwell days shy of the 1:2 objective. In fact, the global demand for AC BCTs would need to drop to 13 in order to achieve a 1:2 ratio. Likewise, until the demand fell to 9 or fewer AC BCTs, the Army could not realize its longer term objective of 1:3.

So if 44 rotational AC BCTs are insufficient to achieve the targeted ratio, how many rotational AC BCTs are required to achieve it? Before we answer this question, we must consider the fact that the sustained AC BCT commitment *(M)* cannot be known with certainty. For example, assume that M might be as little as 15 and as much as 17, with a most likely value of 16. In fact, suppose we can quantify the probabilities of these events as $P(M = 15) = 0.15$, $P(M = 16) = 0.60$, and $P(M = 17) = 0.25$. Based on this probabilistic assessment, we might rephrase our question as "If the Army wanted to have at least a 70% chance of achieving the 1:2 AC BCT BOG:Dwell (or better), how many AC BCTs does it require?"

In order to answer this question, we create an indicator variable I(D), where $D = \frac{N(x-n)}{Mx} - 1$, and $I = 1$ when $D \geq 2$ and $I = 0$ otherwise. Running a parameterized Monte Carlo simulation on *I(D)* where $x = 365$, $n = 40$, and M follows the probability mass function described above, the answer is quickly revealed as 54 AC BCTs. Consider the simulation statistics for $N = 53$ and $N = 54$ in Figure 3 on page 14.

Simply put, if the probability mass function for M holds, then we would need an additional 10 rotational AC BCTs in order to have a 70% chance or

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Available BCT Requirement

	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
6	3.30	2.82	2.44	2.13	1.87	1.64	1.46	1.29	1.15	1.02	0.91	0.81	0.72	0.64	0.56	0.49	0.43
7	3.47	2.97	2.58	2.25	1.98	1.75	1.55	1.38	1.23	1.10	0.99	0.88	0.79	0.70	0.63	0.55	0.49
8	3.60	3.09	2.68	2.34	2.07	1.83	1.63	1.45	1.30	1.16	1.04	0.94	0.84	0.75	0.67	0.60	0.53
9	3.70	3.18	2.76	2.42	2.13	1.89	1.68	1.51	1.35	1.21	1.09	0.98	0.88	0.79	0.71	0.63	0.57
10	3.78	3.25	2.82	2.48	2.19	1.94	1.73	1.55	1.39	1.25	1.12	1.01	0.91	0.82	0.74	0.66	0.59
11	3.84	3.31	2.88	2.52	2.23	1.98	1.77	1.58	1.42	1.28	1.15	1.04	0.94	0.85	0.76	0.68	0.61
12	3.90	3.35	2.92	2.56	2.27	2.01	1.80	1.61	1.45	1.31	1.18	1.06	0.96	0.87	0.78	0.70	0.63
13	3.95	3.40	2.96	2.60	2.30	2.04	1.83	1.64	1.47	1.33	1.20	1.08	0.98	0.88	0.80	0.72	0.65
14	3.98	3.43	2.99	2.63	2.32	2.07	1.85	1.66	1.49	1.35	1.22	1.10	0.99	0.90	0.81	0.73	0.66
15	4.02	3.46	3.02	2.65	2.35	2.09	1.87	1.68	1.51	1.36	1.23	1.11	1.01	0.91	0.83	0.75	0.67
16	4.05	3.49	3.04	2.67	2.37	2.11	1.89	1.69	1.52	1.38	1.24	1.13	1.02	0.92	0.84	0.76	0.68
17	4.08	3.51	3.06	2.69	2.38	2.12	1.90	1.71	1.54	1.39	1.26	1.14	1.03	0.93	0.85	0.77	0.69
18	4.10	3.53	3.08	2.71	2.40	2.14	1.91	1.72	1.55	1.40	1.27	1.15	1.04	0.94	0.85	0.77	0.70
19	4.12	3.55	3.10	2.72	2.41	2.15	1.93	1.73	1.56	1.41	1.28	1.16	1.05	0.95	0.86	0.78	0.71
20	4.14	3.57	3.11	2.74	2.43	2.16	1.94	1.74	1.57	1.42	1.28	1.16	1.06	0.96	0.87	0.79	0.71
21	4.16	3.58	3.13	2.75	2.44	2.17	1.95	1.75	1.58	1.43	1.29	1.17	1.06	0.96	0.88	0.79	0.72
22	4.17	3.60	3.14	2.76	2.45	2.18	1.96	1.76	1.59	1.43	1.30	1.18	1.07	0.97	0.88	0.80	0.72
23	4.19	3.61	3.15	2.77	2.46	2.19	1.96	1.77	1.59	1.44	1.31	1.18	1.07	0.98	0.89	0.80	0.73
24	4.20	3.62	3.16	2.78	2.47	2.20	1.97	1.77	1.60	1.45	1.31	1.19	1.08	0.98	0.89	0.81	0.73

1 : Dwell Ratio; Assuming 45 BCTs (44 Rotational) and 40 day RIP/TOA Length

Figure 2: AC BCT BOG:Dwell Ratios with 44 Rotational BCTs

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Use of Probabilistic Topic Models for Search

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This thesis solves a common issue in search applications. Typically, the user does not know exactly which terms are used in a document he is searching for. Several attempts were made to overcome this issue by augmenting the document model and/or the query. In this thesis, a probabilistic topic model augments the document model. Probabilistic document models are formally introduced and inference methods are derived. It is shown how these models can be used for information retrieval tasks and how a search application can be implemented. A prototype was implemented and the implementation is tested and evaluated based on benchmark corpora. The evaluation provides empirical evidence that probabilistic document models improve the retrieval performance significantly and which preprocessing steps should be made before applying the model.

Biography

Captain Draeger was born on August 20, 1977 in Burg, GDR.

He graduated from High School in 1996 and joined the Army the same year. He was commissioned from 1996 to 2000 in German Army Logistic Forces.

In 2004, he graduated from Federal Armed Forces University, Munich, where he earned a Master of Science degree in Computer Science and was assigned as a first lieutenant to the Center for Transformation of the Federal Armed Forces.

From 2005 to December 2006, he served as an instructor at the Federal Armed Forces Command Support School in Feldafing, Germany, where he taught database technology, UNIX, and IT basics in enlisted and officer programs. From January 2007 to June 2007, he served as the deputy commander of the training effectiveness assessment group at the Command Support School.

In June 2007, Captain Draeger reported to the Naval Postgraduate School to complete a graduate degree in Operations Research.

Captain Draeger is married to Ute Draeger, and they have two children.

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better of achieving a 1:2 BOG:Dwell ratio.

	$I(D)$	
	N=53	N=54
Average	0.147	0.773
Standard Deviation	0.354	0.419
Standard Error	0.011	0.013
Lower Bound of 95% CI for $\mu_{I(D)}$	0.125	0.746
Upper Bound of 95% CI for $\mu_{I(D)}$	0.169	0.800

Figure 3: Parameterized Simulation Statistics (x = 365, n = 40, M ~ Discrete as above)

Conclusion

In the end, one is thing stunningly clear: Given the current policy to cap the force at 45 AC BCTs, the Army will not be able to grow itself back into balance. Moreover, even if the Army had grown to its originally planned 48 AC BCTs, the headline would have been much the same. Indeed, for the foreseeable future the Army's ability to reduce its deployment churn will be directly tied to the demand for its forces, a parameter largely dictated by those who would seek to destroy us and our way of life. Unfortunately, as we are operating in an era of persistent conflict, engaged in what has been rightly dubbed a long war, there appears to be no shortage of such actors, and they do not seem obliged to help.

Addendum

The work in this article was part of a larger presentation to the Vice Chief of Staff of the Army on individual Soldier BOG:Dwell ratios. The theoretical underpinnings presented in this paper served as the basis for a change in direction for the Army in this analysis. This work has been subsequently shared with the entire Army Staff as well as the Chief of Staff of the Army and the Secretary of the Army. It continues to be used to shape Army policy on the deployment of Soldiers and the impacts on them and their Families.

References

Congressional Budget Office. 2009. An Analysis of the Army's Transformation Programs and Possible Alternatives. June, 1-16.

Department of the Army. 2007a. Army Posture Statement: Addendum H (Army Force Generation).

Department of the Army. 2007b (24 August). HQDA Sourcing Policies (BOG and Dwell Time).

Department of the Army. 2008a (10 April). Active Army Twelve Month Boots On The Ground (BOG) Policy.

Department of the Army. 2008b (16 June). Army Campaign Plan 2008 EXORD. A-1.

Department of the Army. 2008c (29 August). MILPER Message Number: 08-219 (AHRC-PL-PN) ARFORGEN Focused Manning.